Performance evaluation of multi-target tracking using the OSPA metric

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Abstract – *Performance evaluation of multi-target tracking algorithms is of great practical importance in the design and comparison of tracking systems. Recently a consistent metric for performance evaluation of multi-object filters (referred to as OSPA metric) has been proposed. In this paper we describe how the OSPA metric can be adapted to evaluate the performance of multi-target tracking algorithms. The main idea is to introduce the track label error into consideration in order to capture the data association performance, in addition to the existing cardinality and localisation errors. The paper demonstrates the proposed method by assessing and comparing the performance of two particle filters for multi-target tracking.*

Keywords: Multi-target tracking, performance evaluation, data association, particle filters.

1 Introduction

Performance evaluation of multi-target tracking algorithms is of great practical importance for system design, tracker comparisons (e.g. in tender assessments), sensor management, etc. Performance evaluation should be distinguished from the performance prediction: the former is performed once the tracking system has been developed and typically involves Monte Carlo simulations; the latter is usually theoretical and can be carried out even before the system has been built.

The topic of multi-target tracker performance evaluation and assessment has been studied extensively in the past [1], [2], [3, Ch.13], [4], [5]. The general methodology assumes that the ground truth is known for tracks. Typically this means that a scenario of a certain level of difficulty is constructed beforehand. Monte Carlo runs are performed for each random output of the signal processing and detection unit, which feeds the tracker with measurements. Typically the measurements are uncertain and ambiguous due to the measurement errors, modelling errors, imperfections of detection (false detections, missed detections), etc. The output of the tracker under assessment needs first be assigned to truth tracks [1]. Once the assignment is made, it is necessary to compute the measures of effectiveness (performance metrics) describing timeliness (e.g. track initiation delay, track overshoot), track accuracy, continuity of tracks (track swaps, track purity [3, Ch.13]), the number and duration of false and divergent tracks, etc. These measures of effectiveness (MoEs) are averaged over Monte Carlo runs in order to provide an assessment of *expected* tracker performance.

The problem a practitioner faces is twofold. The first is how to choose the relevant MoEs and the second is how to combine them into a single score (which is typically required in a tender process). The choice of relevant MoEs is far from clear, as various authors argue in favour of different ones. Combination of MoEs is particularly questionable from the theoretical point of view since the selected MoEs can be correlated. For example, a reduction in the track initiation delay is expected to increase the number of false tracks and vice versa. Another important issue is the "transitive" property of the tracker score resulting from the combination of MoEs. This property can be explained as follows. Consider two trackers, A and B, and suppose that tracker A output is "close" (as indicated by the combined MoE) to the ground truth. If tracker B output is "close" to that of tracker A, can we make the conclusion that tracker B output is also "close" to the ground truth? The answer is positive only if the combined MoE possesses the "transitive" property. It is far from clear if any combination of traditional MoEs would satisfy this property.

In order to overcome the arbitrariness in the selection of MoEs in multi-target tracker performance evaluation, an attempt to define mathematically rigorous metrics for measuring distance between two sets of objects (targets) was reported in [6]. Mathematical metrics naturally satisfy the transitive property for MoEs due to the triangle inequality axiom. The starting point was the Hausdorff metric (distance), which is a metric on sets, but whose performance was found to be unsatisfactory due to its insensitivity to differences in the number of targets (cardinality). Instead Hoffman and Mahler [6] proposed a new multi-object distance (which they refer to as the multi-object Wasserstein distance) which partly fixes the undesirable cardinality behaviour of the Hausdorff metric. Schuhmacher *et al.* [7] subsequently demonstrate a number of shortcomings of the Hoffman-Mahler metric and propose a new consistent metric for sets, which is suitable for multi-target filtering. This distance, referred to as the *optimal subpattern assignment* (OSPA) metric, is shown to completely eliminate most of the shortcomings of the Hoffman-Mahler metric. Schuhmacher *et al.* [7] demonstrated how to use the OSPA metric in the multi-object estimation performance evaluation. This basically concerns only two (instantaneous) aspects of the distance: the cardinality error and the localisation error (the distance in object positions in the state space). For target tracking, however, we require a metric on sets of target trajectories, where by a trajectory we mean a labeled sequences of state vectors over time.

This paper describes how the OSPA distance can be adapted to evaluate the performance of multi-target tracking algorithms. The main idea is to introduce the track label error into consideration, in addition to the existing cardinality and localisation errors. The track label component of the OSPA metric will effectively capture the data association performance. In order to demonstrate the OSPA metric for tracks, the paper also presents a case study where two particle filters for multi-target tracking are assessed and compared. The paper is organised as follows. Section 2 formally introduces the problem and defines the OPSA metric. Section 3 describes how the OSPA metric can be adapted to evaluate multi-target tracking performance. Section 4 presents a comparative case study based on OSPA metric for tracks. The conclusions of the paper are drawn in Section 5.

2 Background

Let $\mathcal{X}$ be an arbitrary non-empty set. A function $d : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_+ = [0, \infty)$ is called a metric on $\mathcal{X}$ if it satisfies the following properties: identity, symmetry and the triangle inequality [8]. If $\mathcal{X} = \mathbb{R}^N$, the function d can be, for example, the Euclidian metric:

$$d(\mathbf{x}, \mathbf{y}) = \left(\sum_{\ell=1}^{N} \left| \mathbf{x}(\ell) - \mathbf{y}(\ell) \right|^p \right)^{1/p} \tag{1}$$

$\mathbf{x}, \mathbf{y} \in \mathbb{R}^N$, $1 \le p \le \infty$, or it can be the Mahalanobis distance $d = \sqrt{(\mathbf{x} - \mathbf{y})^\intercal \mathbf{S}^{-1} (\mathbf{x} - \mathbf{y})}$, where $\mathbf{S}$ is the covariance matrix of the difference $\mathbf{x} - \mathbf{y}$. Throughout the paper $d(\mathbf{x}, \mathbf{y})$ will be referred to as the base distance on the single-object state space $\mathbb{R}^N$.

Consider now two sets $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_m\}$ and $\mathbf{Y} = \{\mathbf{y}_1, \ldots, \mathbf{y}_n\}$, where $n, m \in \mathbb{N}_0 = \{0, 1, \ldots\}$. Vectors $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{Y}$ are taking values from the state space $W \subseteq \mathbb{R}^N$, while sets $\mathbf{X}, \mathbf{Y} \in \mathcal{F}(W)$, where $\mathcal{F}(W)$ represents the set of all finite subsets of W. The OSPA metric is defined as a distance on $\mathcal{X} = \mathcal{F}(W)$, between sets like $\mathbf{X}$ and $\mathbf{Y}$.

Let us introduce the cut-off distance between vectors $\mathbf{x}, \mathbf{y} \in W$:

$$d_c(\mathbf{x}, \mathbf{y}) = \min\{c, d(\mathbf{x}, \mathbf{y})\}, \tag{2}$$

where $c > 0$ is a parameter referred to as the cut-off value.

The OSPA distance of order $1 \le p \le \infty$ with cut-off c is defined for $m \le n$ as follows [7]:

$$D_{p,c}(\mathbf{X}, \mathbf{Y}) = \left[\frac{1}{n} \left(\min_{\pi \in \Pi_n} \sum_{i=1}^{m} \left(d_c(\mathbf{x}_i, \mathbf{y}_{\pi(i)}) \right)^p + (n-m) \cdot c^p \right) \right]^{1/p} \tag{3}$$

where Π_n represents the set of permutations of length m with elements taken from $\{1, 2, \ldots, n\}$. For the case $m > n$, the definition is simply $D_{p,c}(\mathbf{X}, \mathbf{Y}) = D_{p,c}(\mathbf{Y}, \mathbf{X})$. The proof that the OSPA distance is indeed a metric on $\mathcal{X} = \mathcal{F}(W)$ is given in [7].

Note that we can interpret the OSPA distance as a pth order "per-object" error, which is comprised of two components each separately accounting for "localisation" and "cardinality" errors. If $m \le n$, these components are given as:

$$e_{p,c}^{loc}(\mathbf{X}, \mathbf{Y}) = \left[\frac{1}{n} \cdot \min_{\pi \in \Pi_n} \sum_{i=1}^{m} \left(d_c(\mathbf{x}_i, \mathbf{y}_{\pi(i)}) \right)^p \right]^{1/p} \tag{4}$$

$$e_{p,c}^{card}(\mathbf{X}, \mathbf{Y}) = \left(\frac{(n-m)\, c^p}{n} \right)^{1/p}. \tag{5}$$

Otherwise (i.e. if $m > n$), we have $e_{p,c}^{loc}(\mathbf{X}, \mathbf{Y}) = e_{p,c}^{loc}(\mathbf{Y}, \mathbf{X})$ and $e_{p,c}^{card}(\mathbf{X}, \mathbf{Y}) = e_{p,c}^{card}(\mathbf{Y}, \mathbf{X})$. Only for $p = 1$ we can write $D_{p,c}(\mathbf{X}, \mathbf{Y}) = e_{p,c}^{loc}(\mathbf{X}, \mathbf{Y}) + e_{p,c}^{card}(\mathbf{X}, \mathbf{Y})$.

Note that the cut-off parameter c determines the relative weighting given to the cardinality error component against the localisation error component. Smaller values of c tend to emphasize localisation errors and vice versa. Note that c must be in the same units as the localisation error, that is in meters.

The problem we want to solve in this paper is how to extend the OSPA metric to tracks. A track can be seen as a labeled sequence of sets with zero or one element (in the state space), describing the appearance/disappearance and temporal evolution of target dynamics over the surveillance period consisting of K scans. A proposed solution is presented in the next section.

3 New multi-target MoE

In this section we first define the OSPA metric for labeled finite sets. Then we describe how this metric can be used as a MoE for performance evaluation of multi-target tracking.

3.1 Metric for labeled sets

Consider two labeled sets

$$\begin{aligned}\tilde{\mathbf{X}} &= \{(\mathbf{x}_1, s_1), \ldots, (\mathbf{x}_m, s_m)\} \text{ and} \\ \tilde{\mathbf{Y}} &= \{(\mathbf{y}_1, t_1), \ldots, (\mathbf{y}_n, t_n)\},\end{aligned} \tag{6}$$

where as before vectors $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{Y}$ are taking values from the state space $W \subseteq \mathbb{R}^N$ and labels s_i, $(i = 1, \ldots, m)$ and t_j, $(j = 1, \ldots, n)$ are taking values in $\mathbb{N}$. For brevity we will use notation $\tilde{\mathbf{x}} = (\mathbf{x}, s)$ and $\tilde{\mathbf{y}} = (\mathbf{y}, t)$. Effectively we now consider sets on a product space $\mathcal{X} = \mathbb{R}^N \times \mathbb{N}$. The base distance on this set $\mathcal{X}$ is defined as:

$$\tilde{d}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}) = \Big(d(\mathbf{x}, \mathbf{y})^p + d(s,t)^p\Big)^{1/p}, \tag{7}$$

where $d(\mathbf{x}, \mathbf{y})$ was introduced earlier and $d(s,t)$ is the base distance on the set of labels $\mathbb{N}$. We adopt

$$d(s,t) = (1 - \delta[s,t])\alpha \tag{8}$$

where $\delta[i,j]$ is the Kroneker delta, that is $\delta[i,j] = 1$ if $i = j$, and $\delta[i,j] = 0$ otherwise. Parameter $\alpha > 0$ in (8) controls the weighting of the labeling error $d(s,t)$ in relation to the base distance $d(\mathbf{x}, \mathbf{y})$. It is easy to verify that distance $\tilde{d}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})$ defined by (7) and (8) satisfies the mathematical properties of a metric. To prove triangular inequality, let $\tilde{\mathbf{x}} = (\mathbf{x}, s)$,$\tilde{\mathbf{y}} = (\mathbf{y}, t)$,$\tilde{\mathbf{z}} = (\mathbf{z}, u)$ be any three elements of $\mathcal{X} = \mathbb{R}^N \times \mathbb{N}$. Then we have:

$$\begin{aligned}\tilde{d}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})^p &= d(\mathbf{x}, \mathbf{y})^p + \big(\alpha(1 - \delta[s,t])\big)^p \\ &\leq \big(d(\mathbf{x}, \mathbf{z}) + d(\mathbf{z}, \mathbf{y})\big)^p \\ &\quad + \alpha^p\big(1 - \delta[s,u] + 1 - \delta[u,t]\big)^p \\ &\leq d(\mathbf{x}, \mathbf{z})^p + \alpha^p(1 - \delta[s,u]) \\ &\quad + d(\mathbf{z}, \mathbf{y})^p + \alpha^p(1 - \delta[u,t]) \\ &= \tilde{d}(\tilde{\mathbf{x}}, \tilde{\mathbf{z}})^p + \tilde{d}(\tilde{\mathbf{z}}, \tilde{\mathbf{y}})^p\end{aligned}$$

The OSPA metric for labeled sets is defined next with two weighting parameters, $\alpha > 0$ for labeling error and $c > 0$ for cardinality error. For the case $m \leq n$, let us denote the optimal m point subpattern of the points in $\mathbf{Y}$ using cut-off value of c as:

$$\pi^* = \arg\min_{\pi \in \Pi_n} \sum_{i=1}^{m} \big(d_c(\mathbf{x}_i, \mathbf{y}_{\pi(i)})\big)^p, \tag{9}$$

where $d_c(\mathbf{x}, \mathbf{y})$ was defined[1] in (2).

The OSPA for labeled sets $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ such that $m \leq n$ is now defined as:

$$\begin{aligned}D_{p,c,\alpha}(\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}) = &\Bigg[\frac{1}{n}\Big(\sum_{i=1}^{m}\big(d_c(\mathbf{x}_i, \mathbf{y}_{\pi^*(i)})\big)^p \\ &+ \sum_{i=1}^{m} \alpha^p\,(1 - \delta[s_i, t_{\pi^*(i)}]) \\ &+ c^p \cdot (n - m)\Big)\Bigg]^{1/p}\end{aligned} \tag{10}$$

For the case $m > n$,

$$D_{p,c,\alpha}(\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}) = D_{p,c,\alpha}(\tilde{\mathbf{Y}}, \tilde{\mathbf{X}}). \tag{11}$$

The distance $D_{p,c,\alpha}(\tilde{\mathbf{X}}, \tilde{\mathbf{Y}})$ is a metric because it is effectively the OSPA distance (which is proven to be a metric in [7]), with a different base distance in a single-target space, that is equation (1) replaced by (7).

The OSPA for labeled sets comprises of three components, each separately accounting for localisation, cardinality and labeling errors. The localisation and cardinality error components are as in (4) and (5), respectively. The labeling error component is given by:

$$e^{lab}_{p,c,\alpha}(\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}) = \left[\frac{\alpha^p}{n}\sum_{i=1}^{m}\big(1 - \delta[s_i, t_{\pi^*(i)}]\big)\right]^{1/p} \tag{12}$$

for $m \leq n$ and $e^{lab}_{p,c,\alpha}(\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}) = e^{lab}_{p,c,\alpha}(\tilde{\mathbf{Y}}, \tilde{\mathbf{X}})$ if $m > n$. Note that the labeling error component depends on c because the optimal subpattern π^* in (9) is a function of c.

3.2 OSPA metric for tracks

Here we describe how OSPA for labeled sets can be used to evaluate the output of a multi-target tracking system. Let us first adopt the convention that $\tilde{\mathbf{X}}$ refers to the ground truth and $\tilde{\mathbf{Y}}$ to estimated tracks, produced by a tracker. The labels s_i for ground truth tracks are known. The labels for estimated tracks t_j, however, need to be determined before we can apply the OSPA for labeled sets of Sec.3.1.

The solution to the assignment of labels to estimated tracks has to be done globally, using all ground truth tracks and all estimated tracks over K scans. Recall from the introduction that assignment of estimated tracks to true tracks is always a preliminary step in all tracking performance evaluation schemes. To do this step optimally would require to minimise the total distance, both in time and in the state space, between

[1] One can rightly ask why π^* is not determined by minimisation on $\mathbb{R}^N \times \mathbb{N}$, thus taking into account the label mismatch. In the next section we explain that one of the labeled sets has an unreliable label value, determined by a heuristic. Hence we prefer to define π^* on $\mathbb{R}^N$.

all true and estimated tracks. Since this is practically difficult to achieve, we resort to an approximation.

A simple way to assign labels to estimated tracks is by using one of the existing two-dimensional assignment algorithms (Munkres, JVC, auction, etc) [3, p.342]. Let a true track with label s be denoted as $\mathbb{X}_s = (\mathbf{x}^s_{k^s_b}, \ldots, \mathbf{x}^s_{k^s_e})$, where indices k^s_b and k^s_e, such that $1 \leq k^s_b \leq k^s_e \leq K$, denote the beginning and the end of the track, respectively. Similarly, let an estimated track with label t be $\mathbb{Y}_t = (\mathbf{y}^t_{k^t_b}, \ldots, \mathbf{y}^t_{k^t_e})$. The pairwise cost (which is used to form the assignment matrix) of assigning a true track s with an estimated track t can be adopted as:

$$c(s,t) = \begin{cases} \dfrac{\sum_{k=1}^{K} e^s_k\, e^t_k\, \|\mathbf{x}^s_k - \mathbf{y}^t_k\|}{\exp\{\sum_{k=1}^{K} e^s_k\, e^t_k\}}, & \text{if } \sum_{k=1}^{K} e^s_k\, e^t_k > 0 \\ \infty, & \text{otherwise} \end{cases} \quad (13)$$

where $e^\ell_k = 1$ if track $\ell \in \{s,t\}$ exists at time k and zero otherwise. This heuristic favours longer duration estimated tracks to be assigned to true tracks. If an estimated track t has been assigned to true track s, then its label becomes s too. Unassigned estimated tracks are given labels different from all true track labels.

Consider an illustrative example shown in Fig.1, with two true tracks labeled as s_1 and s_2 (thick black lines) and three estimated tracks labeled as t_1, t_2 and t_3 (thinner red, purple and green lines, respectively). After the global assignment of estimated tracks, their labels will become $t_1 = s_1$, $t_2 = s_2$ and t_3 will be assigned a number different from s_1 and s_2. At the point in time when true tracks s_1, s_2 exist, and estimated tracks t_1 and t_3 exist, the cardinality error component will be zero, the localisation error component will be small, but the labeling error component (12) will be $\alpha/2^{1/p}$.

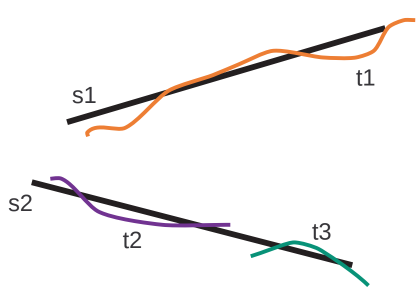


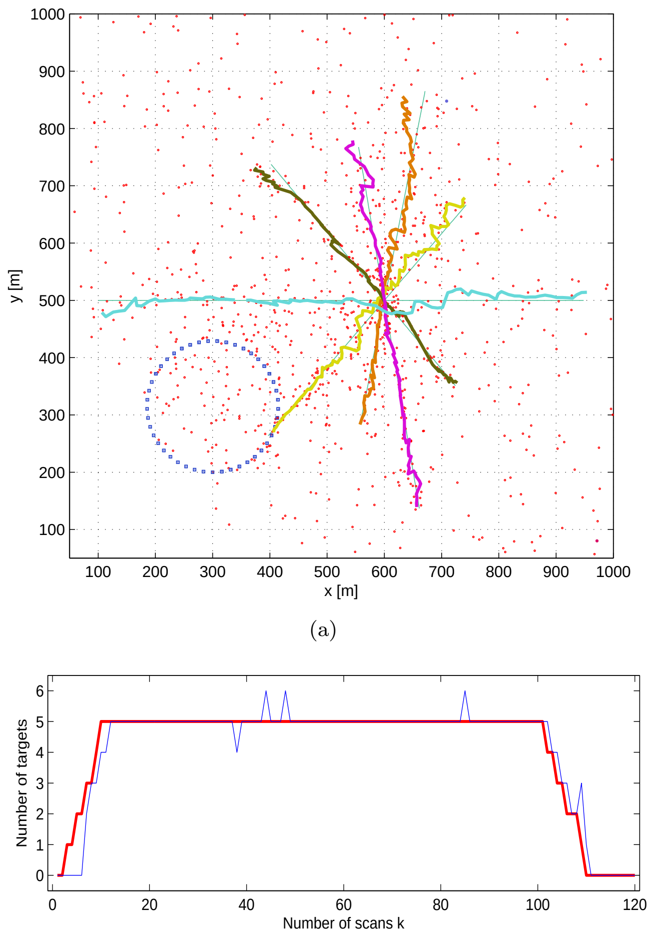
Figure 1: *Example with two true tracks labeled as s_1 and s_2 (thick black lines) and three estimated tracks labeled as t_1 (red), t_2 (purple) and t_3 (green).*

4 Demonstration of OSPA for tracks

This section presents an evaluation of two particle filters for multi-target tracking using the described OSPA metric for tracks.

4.1 Simulation setup

The total duration of the scenario is $K = 120$ scans and consists of five targets moving along the straight lines and crossing at 50 degrees at time $k = 66$, appearing and disappearing at different times. The scenario and the number of targets over time are shown in Fig.2.



(a)

(b)

Figure 2: *The scenario and the output of a single run of the LM-JoTT particle filter: (a) target paths (thin lines represent the ground truth; the blue circle is the observer path; red dots are clutter (false) measurements); (b) cardinality over time (red line is true, blue is the estimate)*

The measurements of target range and bearing are available for tracking. The range measurements are very precise ($\sigma_r = 0.1$m) while bearing measurements are fairly inaccurate (i.e. $\sigma_b = 5$ degrees). This makes the tracking setup very nonlinear: the measurement uncertainty regions are arcs with $\pm 3\sigma$ angular span of 30 degrees. Using Kalman type filters (Extended or unscented Kalman filter), which assume the uncertainty regions to be ellipsoids, would be clearly inappropriate. The clutter is uniformly distributed along the range (from 0 to 1300 m) and bearing ($\pm\pi/4$ rad with respect to the sensor pointing direction). The number of clutter points per scan is Poisson distributed with the mean value of $\lambda = 10$. The probability of detection is $p_D = 0.9$. The effects of finite sensor resolution were ignored.

4.2 Tracking algorithms

Two tracking algorithms are evaluated, using identical framework for estimation and tracking. The only difference is in the way data association is carried out.

Let each target track i at time k be represented by two posteriors:

- the posterior probability of existence $q_k^i = \mathrm{P}(\varepsilon_k|\mathbf{Z}_{1:k})$
- the posterior pdf over the state space $s_k^i(\mathbf{x}) = p_k^i(\mathbf{x}|\mathbf{Z}_{1:k})$

where $\mathbf{Z}_{1:k}$ is the collection of all measurements (from all targets and clutter) accumulated from scan 1 to k; ε_k is the existence, a binary random variable whose transitions are described by the transitional probability matrix:

$$\Pi = \begin{bmatrix} P(\varepsilon_k|\varepsilon_{k-1}) & P(\bar{\varepsilon}_k|\varepsilon_{k-1}) \\ P(\varepsilon_k|\bar{\varepsilon}_{k-1}) & P(\bar{\varepsilon}_k|\bar{\varepsilon}_{k-1}) \end{bmatrix} = \begin{bmatrix} p_S & (1-p_S) \\ p_B & (1-p_B) \end{bmatrix} \tag{14}$$

where p_S is the probability of survival and p_B is the probability of birth. The individual target states are four dimensional vectors, i.e. $\mathbf{x} = [x,\ \dot{x},\ y,\ \dot{y}]^\intercal$, where (x, y) denotes a target position in 2D Cartesian plane, and $(\dot{x}, \dot{y})$ its velocity. Individual target dynamic models are assumed identical, described by the transitional density from time $k-1$ to k denoted by $f_{k|k-1}(\mathbf{x}|\mathbf{x}')$.

The computation of q_k^i and $s_k^i(\mathbf{x})$ is carried out recursively in the Bayesian framework. For this we use the (modified) prediction and update equations of the JoTT filter [9, Sec.14.7], which represent a general version of the integrated PDA [10]. The prediction equations for an existing track described by q_{k-1}^i and $s_{k-1}^i(\mathbf{x})$, where $i = 1, \ldots, n_{k-1}$, are:

$$q_{k|k-1}^i = p_S \cdot q_{k-1}^i \tag{15}$$

$$s_{k|k-1}^i(\mathbf{x}) = \int f_{k|k-1}(\mathbf{x}|\mathbf{x}') s_{k-1}^i(\mathbf{x}') d\mathbf{x}' \tag{16}$$

The update equation for the probability of existence is:

$$q_k^i = \frac{1-\delta_k^i}{1-\delta_k^i\, q_{k|k-1}^i} \cdot q_{k|k-1}^i \tag{17}$$

where

$$\delta_k^i = p_D \left(1 - \sum_{\mathbf{z}\in\mathbf{Z}_k} \frac{\int g_k(\mathbf{z}|\mathbf{x})\, s_{k|k-1}^i(\mathbf{x}) d\mathbf{x}}{\lambda\, c(\mathbf{z})} \right) \tag{18}$$

Here $\mathbf{Z}_k = \{\mathbf{z}_{k,1}, \ldots, \mathbf{z}_{k,\mu_k}\}$ is the collection of all measurements at time k, $g_k(\mathbf{z}|\mathbf{x})$ is the measurement likelihood and $c(\mathbf{z})$ is the clutter spatial density (uniform, see the comments at the end of Sec.4.1).

The update equation for the spatial pdf is given by:

$$s_k^i(\mathbf{x}) = \frac{1 - p_D + p_D \sum_{\mathbf{z}\in\mathbf{Z}_k} \frac{g_k(\mathbf{z}|\mathbf{x})}{\lambda c(\mathbf{z})}}{1-\delta_k^i}\, s_{k|k-1}^i(\mathbf{x}) \tag{19}$$

The described JoTT-like filter works only if there are no conflicts in the assignment of measurements (being effectively a single target tracking algorithm). In order to resolve the possible conflicts in measurement-to-track association (when targets are closed to each other) we apply two modifications of this filter.

LM Tracking. Linear-multitarget (LM) tracking is a very simple and effective way to resolve the conflicts in measurement to track association [11]. The main idea is to modify (increase) the clutter density in eqs.(18) and (19) according to the proximity of neighbouring targets. First let us introduce a prior target i ($i = 1, \ldots, n_{k-1}$) measurement j ($j = 1, \ldots, \mu_k$) density:

$$p_k^i(\mathbf{z}_{k,j}) = \int g_k(\mathbf{z}_{k,j}|\mathbf{x})\, s_{k|k-1}^i(\mathbf{x}) d\mathbf{x} \tag{20}$$

Then compute a priori probability that measurement $\mathbf{z}_{k,j}$ is the detection from the ith target:

$$P_j^i \approx p_D\, p_{k|k-1}^i \frac{\frac{p_k^i(\mathbf{z}_{k,j})}{\lambda c(\mathbf{z}_{k,j})}}{\sum_{\ell=1}^{m_k} \frac{p_k^i(\mathbf{z}_{k,\ell})}{\lambda c(\mathbf{z}_{k,\ell})}} \tag{21}$$

Finally the a priori clutter measurement density of the jth measurement when updating track i is given by:

$$\kappa_k^i(\mathbf{z}_{k,j}) = \lambda\, c(\mathbf{z}_{k,j}) + \sum_{\tau=1,\tau\neq i}^{n_{k-1}} p_k^\tau(\mathbf{z}_{k,j}) \frac{P_j^\tau}{1-P_j^\tau}. \tag{22}$$

Effectively the clutter density of measurement j at target i includes the contribution from all other targets. The update step of the JoTT uses $\kappa_k^i(\mathbf{z}_{k,j})$ of (22) instead of $\lambda c(\mathbf{z}_{k,j})$ in both (18) and (19).

Two-dimensional assignment. Here the idea is to form an assignment cost matrix, where the cost of assigning measurement $j = 0, 1, \ldots, \mu_k$ to track i is computed via a modified version of (17):

$$C(i,j) = -\frac{1-\delta_k^{i,j}}{1-\delta_k^i q_{k|k-1}^i}\, q_{k|k-1}^i \tag{23}$$

where $\delta_k^{i,j} = p_D$ if $j = 0$ (no measurement assigned) and

$$\delta_k^{i,j} = p_D \frac{\int g_k(\mathbf{z}_{k,j}|\mathbf{x})\, s_{k|k-1}^i(\mathbf{x}) d\mathbf{x}}{\lambda\, c(\mathbf{z}_{k,j})} \tag{24}$$

otherwise. Once the best two-dimensional assignment (2DA) is determined, and measurement $j_* \in \{0, 1, \ldots, \mu_k\}$ is assigned to track i, the probability of existence of the track is updated simply as $q_k^i = -C(i, j_*)$. The spatial pdf of track i is updated using the appropriate term of the numerator of (19), that is:

$$s_k^i(\mathbf{x}) = \begin{cases} \dfrac{\frac{p_D\, g_k(\mathbf{z}_{k,j_*}|\mathbf{x})}{\lambda\, c(\mathbf{z}_{k,j_*})} s_{k|k-1}^i(\mathbf{x})}{1-\delta_k^i}, & j_* \in \{1, \ldots, \mu_k\} \\[2ex] \dfrac{(1-p_D) s_{k|k-1}^i(\mathbf{x})}{1-\delta_k^i}, & j_* = 0. \end{cases} \tag{25}$$

The measurements which have not been used in the update step (either P_j^i was too small in the LM method or no track was assigned in the 2DA method) initialise new tracks with the probability of existence equal to p_B. At the end of each cycle of a tracker, the management of tracks is carried out (those with $q_k^i < t_d$ are deleted; those with $q_k^i > t_c$ are reported; here $t_d < t_c$). Finally a union of the newborn tracks and the surviving persisting tracks is formed, followed by a new cycle of the tracker.

Due to the highly non-Gaussian shape of the likelihood function $g(\mathbf{z}|\mathbf{x})$, both trackers are implemented as particle filters [12]. In this framework each track i at time k is specified by a set of weighted random samples $\{\tilde{w}_k^{i,n}, \mathbf{x}_k^{i,n}\}_{n=1}^{\nu}$. The probability of existence and the spatial pdf are thus represented as:

$$q_k^i \approx \sum_{n=1}^{\nu} \tilde{w}_k^{i,n} \leq 1 \tag{26}$$

$$s_k^i(\mathbf{x}) \approx \sum_{n=1}^{\nu} w_k^{i,n}\, \delta(\mathbf{x} - \mathbf{x}_k^{i,n}) \tag{27}$$

where $\delta(\mathbf{x})$ is the Dirac delta function and $w_k^{i,n} = \tilde{w}_k^{i,n} / \sum_{n=1}^{\nu} \tilde{w}_k^{i,n}$ are normalised weights.

4.3 Numerical results

OSPA metric for tracks is applied on $\mathbb{R}^2 \times \mathbb{N}$, that is using only the target position in the two-dimensional Cartesian coordinate system. The base distance was Euclidean as in (1). The OSPA metric parameters were: $p = 1$, $\alpha = 25$m, $c = 50$m.

The filters used a nearly constant velocity motion model with white noise acceleration [13, Sec.6.2.2]. The sampling period was $T = 2$s and process noise intensity $0.3\,\mathrm{m}^2/\mathrm{s}^3$. The number of particles per target was $\nu = 5000$. Fig. 3 shows the OSPA for tracks averaged over 100 Monte Carlo runs of each multi-target tracking algorithm. The two particle filters for multi-target tracking (described above) are referred to as the LM-JoTT-PF and the 2DA-JoTT-PF (a version of the particle filter with the LM method for data association has been published in [14]). Fig. 3 indicates the superior performance of the the LM-JoTT-PF over the 2DA-JoTT-PF.

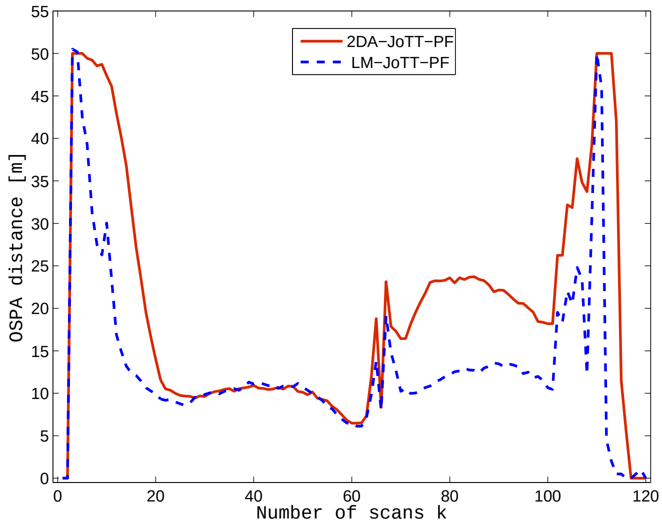


Figure 3: *OSPA metric for tracks (averaged over 100 Monte Carlo runs): 2DA-JoTT-PF (red solid line and LM-JoTT-PF (blue dashed line)*

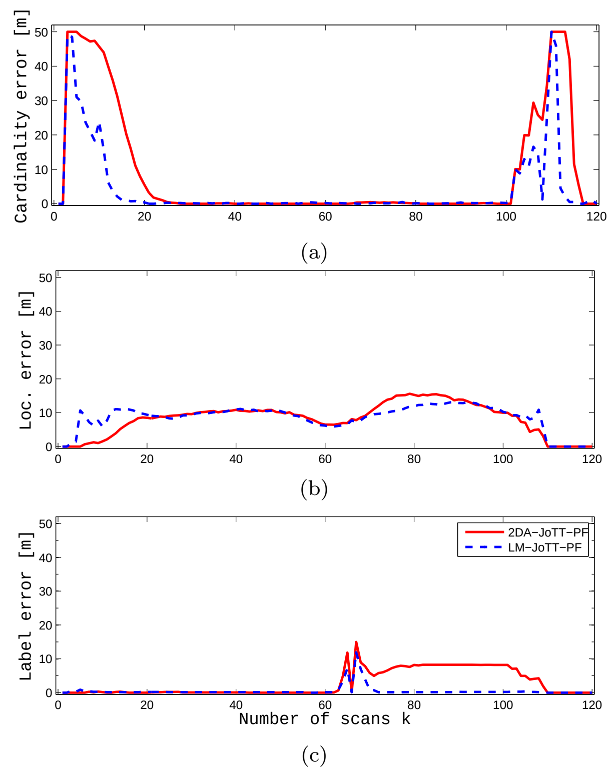


Figure 4: *OSPA distance components: (a) cardinality, (b) localisation and (c) labeling errors*

Using the error components of OSPA metric we can determine more precisely the sources of the difference in performance. This is illustrated in Fig.4 which displays the cardinality, localisation and labeling error component of the total OSPA distance of Fig.3. Since $p = 1$, the OSPA distance equals the sum of the three error components.

Observe that initially (in the first 20 scans) the OSPA distance is large mainly due to the cardinality error. This is a consequence of the fact that both filters need a few scans to establish the tracks. The cardinality error of the LM-JoTT-PF returns to zero quicker, indicating that its calculations of probability of existence are more responsive to the appearance of new targets. After $k = 20$, the OSPA metric is dominated by the localisation error. This error component appears to be similar in both trackers, which is to be expected since they are both constructed on the basis of the same single-target filter (JoTT) and the tracks in this interval are well separated in the state space. Note also that during this interval the OSPA distance gradually reduces with time, reaching its minimum just before $k = 66$, when the tracks cross. Crossing of tracks is reflected in a sudden jump of OSPA metric just after $k = 66$, which is attributed to the labeling error, see Fig.4.(c). In the interval from target crossing to approximately $k = 100$ (when true tracks start to disappear one by one), the labeling error of the LM-JoTT-PF is much smaller than the labeling error of the 2DA-JoTT-PF. Finally after $k = 100$, the cardinality error again dominates the OSPA distance since both trackers need a few scans to terminate/delete the tracks. Again the LM-JoTT-PF is more responsive to the cardinality change due to the disappearance of targets.

5 Conclusions

The paper presented an adaptation of the OSPA metric, previously defined for multi-object filtering, to measure the effectiveness of multi-target tracking systems. The new performance evaluation metric, referred to as the *OSPA for tracks*, is a consistent, intuitively reasonable and mathematically rigorous metric for measuring the distance between two sets of labeled objects. OSPA for tracks has two parameters which determine the relative weighting of the cardinality and the miss-labeling errors against the localisation error. The paper demonstrated the application of OSPA for tracks to performance evaluation and comparison of two multi-target tracking particle filters. We observed that the two different methods of data association produced remarkably different errors in cardinality and labeling, while the localisation errors appear to be similar. The evaluation results obtained using the OSPA metric for tracks appear to be in a good agreement with our intuition.

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